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Economics of Wind Generated Power

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The method explained in the paper considers the various economic and engineering variables that are necessary to properly plan a natural energy system. The goal of the method is to produce an optimum size range for the system and to provide a maximum allowable cost based on area of the collector that can be spent while still retaining an economic advantage. The method uses data on the power requirement of the installation as well as estimates of the finance cost and expected increases in the cost of conventional fuel. The technique includes systems operating in parallel with conventional power sources and excludes any systems with energy storage requirements.

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Economics of Wind Generated Power

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INTRODUCTION

The application of wind, solar, and other natural energy forms to fill our energy needs is a matter of both engineering and economics. Too little emphasis is placed on the economics of such systems in an effort to achieve low cost energy with maximum engineering efficiency. The end result may be workable from a technical aspect but may be too costly to be truly practical. Therefore, there exists a need for a systematic planning approach to estimate the optimum size and cost for a given installation given basic engineering and economic data.

Such a planning method should predict the largest area of energy collector that should be constructed and should also predict the maximum cost per unit area that can be expended in order for the system to pay for itself over a given period of time. Since most natural energy collectors, such as wind turbines, solar panels, or hydro generators, can be priced at a cost per area figure, the method should be applicable to a wide variety of energy sources. Any user interested in applying such a system to fill a particular requirement should use a balanced planning method to avoid constructing too large a system at a cost that is too high to produce any saving in energy cost.

The method explained in this report will relate the value of the natural resource system in dollars per area to the cost of conventional energy sources in dollars per kwh. Only wind energy will be considered here, although the logic is the same for any system whose cost and annual energy output is related to its area.

APPLICABLE EQUATIONS

The basic equation is that representing the total cost of a wind energy conversion system (WECS) in terms of total fixed cost and operating expenses. With time not considered, this is

$$\left[\frac{\text{Fixed Cost}}{\text{Area}} \times \text{Area}\right] + \left[\frac{\text{Operating Cost}}{\text{Area/Year}} \times \text{Area} \times \text{Years}\right] \quad (1)$$

This equation is valid for only one year because no consideration is made for equipment financing or increases in operating cost due to inflation. Factors must be added to these terms to give a true cost over n years in terms of present day dollars.

The expense of financing can be handled using the standard formula for payments on a loan of n periods at interest rate I_m (subscript m for money). The payment formula is (1):[1]

$$\text{pmt} = \text{PV} \cdot \left[\frac{I_m}{1-(1+I_m)^{-n}}\right] \quad (2)$$

The value of PV is the present day cost of the equipment to be financed. Thus, the total amount spent on the equipment is the periodic payment times the number of periods. For a cost per unit area, F_W (subscript w for wind), the total cost of equipment including financing becomes:

$$F_w \cdot \underbrace{(n) \cdot \left[\frac{I_m}{1-(1+I_m)^{-n}}\right]}_{A'} \quad (3)$$

The term grouped as A' is the factor that will increase the value of F_W to reflect the additional cost of financing.

The periodic operating cost per unit area (P_W) will also require adjustment to convert the total maintenance and other annual costs to be spent over n years to present-day

[1] Underlined numbers in parentheses designate References at end of paper.

values. This cost is the sum of each annual cost over the total period with each year taking an increase of I_i percent (subscript i for inflation). The total operating cost becomes:

$$\sum_{1}^{n} P_w \cdot (1+I_m)^{n-1} \qquad (4)$$

This situation is similar to a sinking fund in which a fixed amount of money is deposited each period at a fixed rate of interest to end up with a predetermined amount at the end of n years. In this case, the money to be spent in the future is inflating rather than gaining interest; the end result is the same. Therefore, the sinking fund formula can be used to account for the effect of inflation on the periodic operating cost per unit area (1).

$$\frac{\text{Total Operating Cost}}{\text{Area}} = P_w \cdot \underbrace{\left[\frac{(1+I_i)^{n-1}}{I_i}\right]}_{B'} \qquad (5)$$

The term grouped as B' is the factor that increases the value of P_W to account for inflation. This same reasoning will later be used to account for future rising fuel costs per unit of energy.

Now that the time inclusive factors have been presented, the total cost per area of the WECS over n years in present-day dollars can be expressed by:

$$\frac{TC_n}{\text{Area}} = F_w \left[\frac{n \quad I_m}{1-(1+I_m)-n}\right] + P_w \left[\frac{(1+I_i)^n - 1}{I_i}\right] \qquad (6)$$

or, in shorter notation

$$\frac{TC_n}{\text{Area}} = F_w A' + P_w B' \qquad (6a)$$

Other necessary equations are those relating the energy available per area of the wind at the site in question. Golding (2) discusses this in regard to wind turbines and evaluates the effect of wind data on total energy estimates. It was concluded that accurate information is difficult to obtain without lengthy, precise measurements. However, for estimation purposes, a corrected average wind speed and the number of hours per year that speed is available can serve as an approximation of the total energy available at a given location.

This corrected wind speed is necessary because the energy available varies with the cube of wind velocity (3). Thus, a doubling of wind speed represents an eight fold increase in energy. The minimum information necessary to estimate available energy is the average wind speed and its annual duration in hours. A better estimate can be obtained with information on velocity distribution over a long period of time. This information, however, will generally not be available (2).

Using only average wind information, the energy can be approximated using the following formula (2, 3):

$$\frac{\text{Energy}}{\text{Area-Year}} = .4(K) \cdot (Av)^3 \cdot (H_w) \qquad (7)$$

where:

- Av = corrected wind velocity (usually taken as 1.15 x mean wind speed unless more detailed information is available)
- H_W = hours per year mean wind is available
- K = conversion constant (values for K in various units given at the end of this report) for power in kw and area in ft^2 $K=5.3 \times 10^{-6}$

The constant of 0.4 is necessary to derate the total wind energy present to a value that can be extracted under actual conditions. It is the product of the mechanical efficiency of the WECS (75 percent) (4) and the Betz coefficient of 0.593 (2). The Betz coefficient is the theoretical maximum fraction of power that can be extracted by a wind turbine under ideal conditions.

Now that the amount of energy per area is known, the dollar value of this energy can be determined by comparing it to the cost of conventional energy over some period of time. The conventional source can be a power company or private generation equipment. The dollar value of the WECS energy is the kwh obtained in equation (7) times the energy cost per kwh of the conventional source. If VE is the value of the energy obtained per unit area, it is given by

$$VE = (0.4) \cdot (K) \cdot (Av)^3 \cdot (H_w) \cdot FC \qquad (8)$$

where FC is the fuel cost or energy cost of the conventional source in \$/kwh. The units of VE are \$/Area. Equation (8) is only valid for the first year, and a total value over n years is necessary to determine the true value of the WECS. It is, therefore, necessary to add a factor to the fuel cost, FC, to account for the number of years and the increase in fuel cost I_f (subscript f for fuel) over n years. This is

GENERAL CURVE OF

MAXIMUM F_w VERSUS TIME

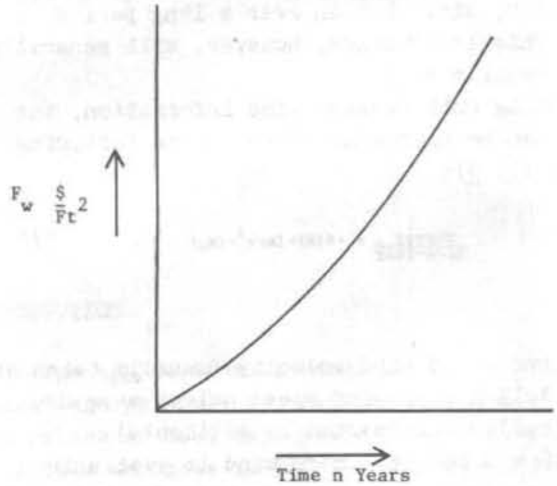

Fig. 1

similar to the adjustment made to the operating cost mentioned earlier. Thus, the total value of WECS energy over n years becomes:

$$VE_n = (0.4)\cdot(K)\cdot(Av)^3\cdot(H_w)\cdot(FC)\cdot\underbrace{\left[\frac{(1+I_f)^n - 1}{I_f}\right]}_{C'} \qquad (9)$$

The term grouped as C' is the factor that accounts for increases in fuel costs over n years. For simplicity, let K' equal the energy per unit area defined as:

$$K' = (0.4)\cdot(K)\cdot(Av)^3\cdot H_w \qquad (10)$$

so that VE_n is defined as:

$$VE_n = K'\cdot(FC)\cdot(C') \qquad (11)$$

In order for the WECS to be economically feasible, the total cost of purchase and operation must equal to or lower than the equivalent value of the energy it produces. This is expressed as:

$$F_w\cdot(A') + P_w\cdot(B') \leq K'\cdot(FC)\cdot(C') \qquad (12)$$

where A', B', and C' are the time variant factors presented in the foregoing.

P_W is normally expressed as a percentage P percent of F_W (4); taking this into account and solving for F_W produces:

$$F_w \leq \frac{FC\cdot(K')\cdot(C')}{A'+P\%\cdot(B')} \qquad (13)$$

GENERAL CURVES OF SAVINGS

VERSUS TIME FOR VARIOUS

FRACTIONS OF F_w MAXIMUM

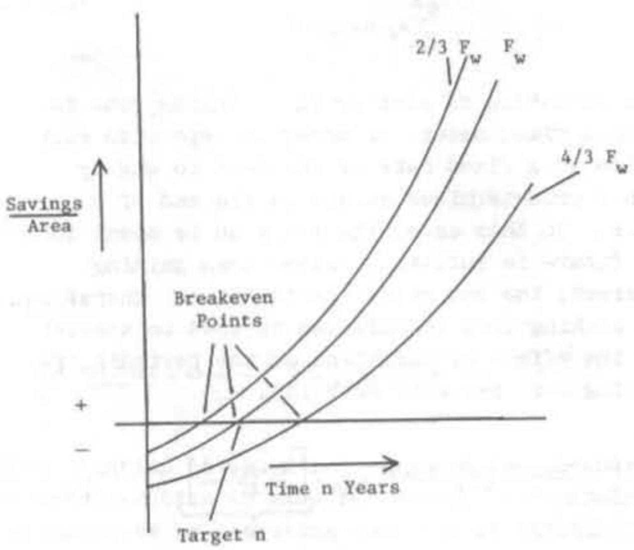

Fig. 2

The F_W of equation (13) is the maximum that can be spent on the WECS per unit area in order for the system to pay for itself in fuel saved over n years. A graph of F_W versus n is shown in Fig. 1.

A system built at the maximum allowable F_W will break even in n years. This is because the total cost of the WECS will have equaled the value of conventional energy it replaced. To obtain a saving in cost per kwh over n years, the cost of the WECS must be less than the maximum of equation (13).

The savings over n years is the difference in the total WECS cost and the value of the energy it produces. This is given by:

$$\frac{Savings}{Area} = K'\cdot(FC)\cdot(C') - F_w\cdot(A') - P_w\cdot(B') \qquad (14)$$

A graph of the savings at various F_W for a general case is shown in Fig. 2. Note that for F_W less than the maximum, the break even point is earlier than the target date at n years energy produced between the break even point and the target date is essentially free. All energy produced after the break even point is free except for maintenance charges as long as the unit can be operational. However, n will usually be taken as the life of the WECS to minimize yearly expenditures by spreading the capital investment over as long a period as possible. Therefore, an F_W greater than the maximum will always produce a system that cannot pay for itself over its useful lifetime.

What size unit to build is more a matter of energy required rather than economics. A given installation will have an average power requirement (PR) that represents the typical load placed on any source. If the maximum power requirement is not much above the average and is fairly infrequent, it is usually beneficial to design the WECS to fill this average requirement (PR) rather than the maximum. A WECS designed to deliver a peak output much above PR will waste power capability and the money spent to build it (3). Excess power that cannot be used cannot be used to calculate fuel savings although the cost of building this capability is included in F_W and P_W. This will reduce the savings in equation (14) and will lengthen the break-even period. It is better to design for the average and obtain additional power from other sources when it is needed.

The area of the rotor (A_W) is given by:

$$\frac{A_W \cdot K'}{H_W} \leq PR$$

or (15)

$$A_W \leq \frac{PR \cdot H_W}{K'}$$

The A_W of equation (15) will produce power equal to PR for H_W hours and decreasing amounts of power the remainder of the year.

ASSUMPTIONS

In applying this method, several assumptions must be made.

1 All energy produced by the WECS is immediately usable by the installation. The additional cost of energy storage greatly increases F_W to the point where long periods of time are necessary to reach a break even point. The effect on the method is to limit its application to installations where energy use is fairly constant over all hours of the day.

2 The WECS is acting in parallel as a supplement to a conventional source that supplies power during low winds and calm periods. Most installations require power on demand and cannot wait for favorable weather. The storage system excluded in assumption 1 connected to a much larger WECS would be necessary if the WECS were to produce all necessary energy. This would further increase the capital investment and other costs and would increase the payback period beyond a reasonable number. The effect is to limit the method to applications where the WECS is not the only source of energy.

3 The values of I_m, I_i, and I_f are constant with time. The loan interest rate will almost always be a known constant at n = 0, but the other rates must be estimated at a constant rate for the period of time to be considered. The effect is to decrease reliability of the method with increasing n. The value of I_f is especially important and will probably be the most difficult to estimate for long periods of time. Government sources should be able to provide established figures for at least 10 years.

4 The value of n does not exceed the expected lifetime of the WECS. If the unit is unoperational for the later periods of n, it cannot produce any energy to offset the capital payments that would still be in progress. A lifetime less than n would give an incorrect F_W that is too high. A more reliable approach is to choose n less than the lifetime of the unit so that the WECS will have additional time to produce free energy after it is paid for and only operational expenses remain. Also, a shorter n will increase reliability of the interest rates and will leave the WECS with a salvage value at the end of n years.

METHOD OF APPLICATION

Anyone interested in applying wind power to provide an alternate energy source and to lower energy cost can apply this method with a minimum of input data.

The first value to be determined is A_W, the maximum area that can be utilized. This requires basic information on the installation's power requirement and the local wind data as discussed earlier. The various factors A', B', and C' are then calculated for the length of time to be considered, usually the WECS lifetime. This enables a maximum F_W to be calculated and compared to information on current construction costs for year zero. Only when the unit can be built for less than the maximum F_W will the WECS be feasible from an economic standpoint. A look at Fig. 1 shows that increasing time allows higher F_W which means more expensive units are practical. Too long of a period may be avoided, particularly on large installations, due to uncertainties about the estimate of I_f. If a construction cost less than F_W maximum can be realized, the payback period can be found by plotting the savings of equation (14) against time. The point where the curve is the break-even point, and energy produced after that point will be essentially cost free. As can be seen in Fig. 2, a longer operating period greatly increased total savings although the system may operate at a loss for a number of years.

CONCLUSION

The optimum size and cost of a WECS is a function of many variables. The effects of the important variables on F_W are summarized in the following table:

Effect on F_W – ↑ Higher or ↓ Lower

Variable	Increase	Decrease
I_m	↓	↑
I_i	↓	↑
I_f	↑	↓
K'	↑	↓
FC	↑	↓
P%	↓	↑

The optimum situation is a construction cost far below the maximum F_W in a geological area with a higher Av and H_W. Lower financing rates, as with government bonds, and lower expected inflation rates against higher fuel costs and index are also encouraging to WECS construction.

The method is easily converted to other natural energy forms by changing the input data to K'. This variable is the energy per area per year that can practically be collected at the site. The appropriate information on yearly sunlight or water flow would be necessary to convert K' for solar or hydro collection. The remainder of the equations remain unchanged for other energy forms.

EXAMPLES

Consider a homeowner who desires to add the cost of a wind turbine to a new home mortgage. The yearly energy requirement of the home is 26,000 kwh (5). An average load of 3 kw is expected and the mortgage is to be for 20 years. The local average wind is 13 mph for 4200 hr per year (6). The load interest rate is 8 percent, and the inflation rate and fuel increase index are taken as the published figures of 4.5 and 15 percent, respectively (4). Maintenance is expected to be 3 percent of the fixed cost (4). Current electricity cost is 4.5 ¢/kwh (5).

First determine the area to be considered. From equations (10) and (15):

$$K' = 0.4\cdot(5.3\times10^{-6})\cdot(1.15\cdot13)^{3}\cdot(4200)$$

$$K' = 29.75\ \frac{\text{KWH}}{\text{Ft}^2\text{-Yr}}$$

$$A_W = \frac{3\cdot 4200}{29.75} = 425\ \text{Ft}^2$$

Next, calculate A', B', and C' for n=20 years

$$A' = 20\cdot\left[\frac{0.08}{1-(1.08)^{-20}}\right] = 2.04$$

$$B' = \frac{(1.045)^{20}-1}{0.045} = 31.37$$

$$C' = \frac{(1.15)^{20}-1}{0.15} = 102.44$$

The maximum cost per area at 20 years is then calculated from equation (13):

$$F_n \leq \frac{(0.045)\ (29.75)\ (102.44)}{2.04+(0.03)\ (31.37)}$$

$$F_n \leq 46.00\ \frac{\$}{\text{Ft}^2}$$

This is a fairly high figure due primarily to the long term of 20 years. However, a WECS that will last 20 years will be more expensive than one that will last only 10 years or less. Many of smaller units available today (1977) are advertised as having a lifetime of 10 years. Even the best units are only 30 to 35 $/ft² installed, so this homeowner can afford the best unit up to his maximum area of 538 ft². His saving over 20 years from equation (14) is

$$\frac{\text{Saving}}{\text{Ft}^2} = (29.75)\cdot(0.045)\cdot(102.44)-(37.5)\cdot(2.04)-(0.03)\cdot(35.0)\cdot(31.37)$$

$$= 25.35\ \frac{\$}{\text{Ft}^2}$$

For a total of 425 ft², the total savings over the life of the unit (20 years) is

$$25.35 \times 425 = \$10{,}775$$

This may seem like a large figure, but it must be realized that one kwh will cost much more in 20 years. In this case, the cost per kwh in 20 years is $.65 if the 15 percent increase is accurate. At that rate, $10,775 is not so large when compared to the energy it actually represents. Over 20 years, the total energy produced by the WECS is

The home would have used 26,000 x 20 or 520,000 kwh, so the WECS provides 49 percent of the homes energy and saves money as well.

As a second example, consider a chemical plant desiring wind energy to supplement electric power in process heat generation. The yearly energy requirement is 1×10^6 kwh with a typical load of 100 kw. Local winds are 17 mph for 4900 hr per year. Maintenance is expected to be 4 percent of the fixed cost with electricity currently costing 2.75 ¢/kwh. Use the same economic information given in the previous example with n = 15 years.

$$K' = 0.4 \cdot (5.3 \times 10^{-6}) \cdot (1.15 \cdot 17)^3 \cdot (4900)$$

$$K' = 77.62 \ \frac{KWH}{Ft^2\text{-}Yr}$$

$$A_W = \frac{100 \cdot 4900}{77.62} = 6312 \ Ft^2$$

$$A' = 15 \cdot \frac{0.08}{1-(1.08)^{-15}} = 1.75$$

$$B' = \frac{(1.045)^{15}-1}{0.045} = 20.78$$

$$C' = \frac{(1.15)^{15}-1}{0.15} = 47.58$$

$$F_W \leq \frac{(0.0275) \cdot (77.62) \cdot (47.58)}{1.75 + (0.04)\ (20.78)}$$

$$F_W \leq 39.25 \ \frac{\$}{Ft^2}$$

Large wind turbines are currently (1977) being estimated between 35 and 40 dollars per square foot (4). This example represents a marginal installation that is caused by the relatively low energy cost for an industrial user. Note also that n is only 15 years, causing a lower index to increase the present-day fuel cost which is already low. Although this installation may not produce direct energy savings, the alternate energy source it provides at no increase in cost per kwh may be reason enough to build it if the future availability of conventional electricity is a questionable subject.

As a last example consider a power plant where it is desirable to use wind energy as an electric power source for the utility grid. The company uses No. 6 fuel oil in the steam generators at a cost of 0.0169 \$/kwh output (5). The output of the plant is 3.5×10^9 kwh per year with a typical load of 400 MW. Local winds are 18 mph for 4500 hr per year. A lower finance rate of 6 percent is available from government sources, although I_f and I_1 are the same as in the previous examples. A lifetime of 15 years is necessary for government financing, and the periodic maintenance is expected to be 5 percent due to the complex control systems such as grid feeding generators require. Continuing as before,

$$K' = 0.4 \cdot (5.3 \times 10^{-6}) \cdot (1.15 \cdot 18)^3 \cdot (4500)$$

$$K' = 84.62 \ \frac{KWH}{Ft^2\text{-}Yr}$$

$$A_W = \frac{(400 \times 10^3) \cdot (4500)}{84.62} = 21.3 \text{ million } Ft^2$$

This is a huge area, and the maximum A_W would probably not be utilized due to financial as well as engineering problems of such a large installation. The calculation of F_W is:

$$A' = \frac{20 \cdot 0.06}{1-(1.06)^{-20}} = 1.74$$

$$B' = \frac{(1.045)^{20}-1}{0.045} = 31.37$$

$$C' = \frac{(1.15)^{20}-1}{0.15} = 102.44$$

$$F_W \leq \frac{(0.0169)\ (84.62)\ (102.44)}{1.74 + 0.04\ (31.37)}$$

$$F_W \leq 48.91 \ \frac{\$}{Ft^2}$$

An actual cost less than F_W could probably be realized, especially considering the large area to be utilized. The 21 million square feet given as the optimum is too large to be practical, although several million square feet could be obtained in several large units.

Suppose that 3 million square feet is to be built at \$40/ft². This can be accomplished by 15 units with rotor diameters of 250 ft, less than the largest currently considered feasible from a technical standpoint (4). This represents a fixed cost of 120 million dollars, a realistic sum for such a project. The savings of the venture can be calculated as before:

$$\frac{\text{Savings}}{\text{Area}} = (84.62) \cdot (0.0169) \cdot (102.44) - 40 \cdot (1.74) - (.04) \cdot (40) \cdot (31.37)$$

$$\frac{\text{Savings}}{\text{Area}} = 26.67 \ \frac{\$}{Ft^2}$$

For the 3 million square feet, the total savings is $(3 \times 10^6)\ (26.67) = 80$ million dollars. This saving is above the cost of the wind turbines and is in terms of present-day dollars.

CLOSING

As can be seen in the examples, the actual calculation of the optimum areas and costs are easy when the information required is available. The gathering of the input data is the most difficult part, although increasing use of wind and other natural energy resources should provide more accurate data from which better and more reliable estimates can be made. For the present, the method should be used in conjunction with good engineering and economic judgment to determine the best natural energy conversion system for each specific installation.

APPENDIX

Values of K for various units (2):

Power	Area	Velocity	K
kw	ft^2	mph	5.3×10^{-6}
kw	ft^2	knots	8.1×10^{-6}
hp	ft^2	mph	7.1×10^{-6}
watt	ft^2	fps	1.7×10^{-3}
kw	$meter^2$	meter/sec	6.4×10^{-4}
kw	$meter^2$	kilometer/sec	1.4×10^{-5}

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